Expressing Implications

q if p, p is sufficient for q

p	q	$p \Rightarrow q$	
true	true	true	
true	false	false	
false	true	true	
false	false	true	

p: snow storm

q: cancel class

q unless -p

р	q	$p \Rightarrow q$		
true	true	true		
true	false	false		
false	true	true		
false	false	true		

p only if q, q is necessary for p

p	q	$p \Rightarrow q$	
true	true	true	
true	false	false	
false	true	true	
false	false	true	

Logical Quantifiers: Examples

$$\forall \ i \bullet i \in \mathbb{N} \Rightarrow i \geq 0$$

$$\forall i \bullet i \in \mathbb{Z} \Rightarrow i \geq 0$$

$$\forall$$
 i, j \bullet i \in \mathbb{Z} \land j \in \mathbb{Z} \Rightarrow i $<$ j \lor i \gt j

$$\exists i \bullet i \in \mathbb{N} \land i \geq 0$$

$$\exists i \bullet i \in \mathbb{Z} \land i \geq 0$$

$$\exists i, j \bullet i \in \mathbb{Z} \land j \in \mathbb{Z} \land (i < j \lor i > j)$$

Logical Quantifiers: Examples

How to prove $\forall i \bullet R(i) \Rightarrow P(i)$?

How to prove $\exists i \bullet R(i) \land P(i)$?

How to disprove \forall i \bullet R(i) \Rightarrow P(i) ?

How to disprove $\exists i \bullet R(i) \land P(i)$?

Prove/Disprove Logical Quantifications

• Prove or disprove: $\forall x \bullet (x \in \mathbb{Z} \land 1 \le x \le 10) \Rightarrow x > 0$.

• Prove or disprove: $\forall x \bullet (x \in \mathbb{Z} \land 1 \le x \le 10) \Rightarrow x > 1$.

• Prove or disprove: $\exists x \bullet (x \in \mathbb{Z} \land 1 \le x \le 10) \land x > 1$.

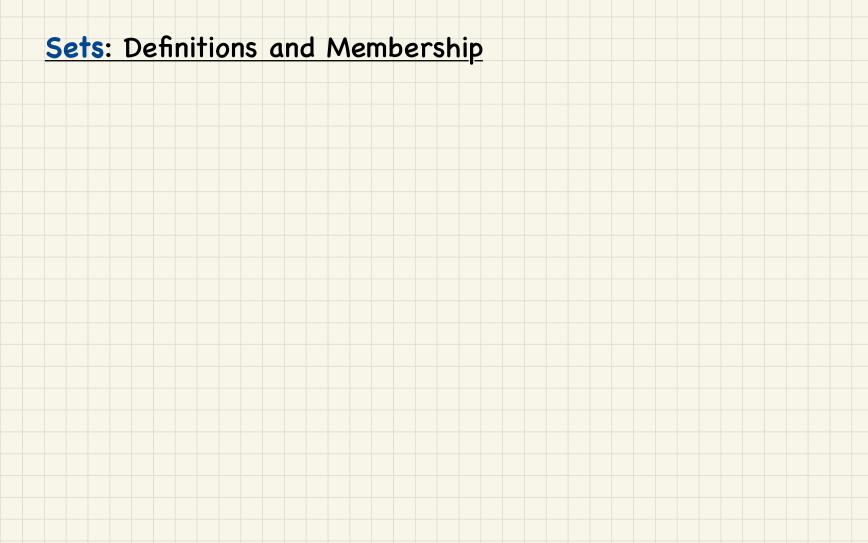
• Prove or disprove that $\exists x \bullet (x \in \mathbb{Z} \land 1 \le x \le 10) \land x > 10$?

Logical Quantifications: Conversions

R(x): x ∈ 3342_class P(x): x receives A+

$$(\forall X \bullet R(X) \Rightarrow P(X)) \Leftrightarrow \neg(\exists X \bullet R \land \neg P)$$

$$(\exists X \bullet R \land P) \Leftrightarrow \neg(\forall X \bullet R \Rightarrow \neg P)$$



Relating Sets



Sets: Exercises

Set membership: Rewrite e ∉ S in terms of ∈ and ¬

Find a common pattern for defining:

- 1. = (numerical equality) via ≤ and ≥
- 2. = (set equality) via \subseteq and \supseteq

$$S = \{1, 2, 3\}, T = \{2, 3, 1\}, U = \{3, 2\}$$

	S		Τ		U	
S	<u>_</u>	C	<u>_</u>	C	<u>_</u>	C
T	⊆	С	⊆	С	⊆	C
U	⊆	С	⊆	C	⊆	C

Is set difference (\) commutative?

Combinations: Formula and Interpretation

Power Set

Calculate the power set of {1, 2, 3}.

Given a set S, formulate the cardinality of its power set.

Cardinality of Power Set: Interpreting Formula

- Calculate by considering subsets of various cardinalities.
- Calculate by considering whether a member should be included.

Set of Tuples

Given n sets S_1, S_2, \ldots, S_n , a *cross/Cartesian product* of theses sets is a set of n-tuples.

Each n-tuple (e_1, e_2, \ldots, e_n) contains n elements, each of which a member of the corresponding set.

$$S_1 \times S_2 \times \cdots \times S_n = \{(e_1, e_2, \dots, e_n) \mid e_i \in S_i \land 1 \leq i \leq n\}$$

Example: Calculate {a, b} **X** {2, 4} **X** {\$, &}

Set of Possible Relations

- Set of possible relations on S and T:
- Dedicated symbol for set of possible relations on S and T:
- Declare that set r is a relation on S and T:

Example: Enumerate all relations on {a, b} and {2, 4}.

Hint: How many?