Expressing Implications

p: snow storm
q: cancel class

q if p, p is sufficient for q

p	q	$p \Rightarrow q$
true	true	true
true	false	false
false	true	true
false	false	true

q unless ¬p

p	q	$p \Rightarrow q$
true	true	true
true	false	false
false	true	true
false	false	true

p only if q, q is necessary for p

p	q	$p \Rightarrow q$
true	true	true
true	false	false
false	true	true
false	false	true

Logical Quantifiers: Examples

$$\forall i \bullet i \in \mathbb{N} \Rightarrow i \geq 0$$

$$\forall i \bullet i \in \mathbb{Z} \Rightarrow i \geq 0$$

$$\forall i, j \bullet i \in \mathbb{Z} \land j \in \mathbb{Z} \Rightarrow i < j \lor i > j$$

$$\exists i \bullet i \in \mathbb{N} \land i \geq 0$$

$$\exists i \bullet i \in \mathbb{Z} \land i \geq 0$$

$$\exists i, j \bullet i \in \mathbb{Z} \land j \in \mathbb{Z} \land (i < j \lor i > j)$$

Logical Quantifiers: Examples

How to prove $\forall i \bullet R(i) \Rightarrow P(i)$?

How to prove $\exists i \bullet R(i) \land P(i)$?

How to disprove $\forall i \bullet R(i) \Rightarrow P(i)$?

How to disprove $\exists i \bullet R(i) \land P(i)$?

Prove/Disprove Logical Quantifications

- Prove or disprove: $\forall x \bullet (x \in \mathbb{Z} \land 1 \leq x \leq 10) \Rightarrow x > 0.$

- Prove or disprove: $\forall x \bullet (x \in \mathbb{Z} \land 1 \leq x \leq 10) \Rightarrow x > 1.$

- Prove or disprove: $\exists x \bullet (x \in \mathbb{Z} \land 1 \leq x \leq 10) \land x > 1.$

- Prove or disprove that $\exists x \bullet (x \in \mathbb{Z} \land 1 \leq x \leq 10) \land x > 10$?

Logical Quantifications: Conversions

$R(x)$: $x \in$ 3342_class

$P(x)$: x receives A+

$$(\forall X \bullet R(X) \Rightarrow P(X)) \Leftrightarrow \neg(\exists X \bullet R \wedge \neg P)$$

$$(\exists X \bullet R \wedge P) \Leftrightarrow \neg(\forall X \bullet R \Rightarrow \neg P)$$

De Morgan

Sets: Definitions and Membership

Relating Sets

Relating Sets: Exercises

Sets: Exercises

Set membership: Rewrite $e \notin S$ in terms of $\in$ and $\neg$

Find a common pattern for defining:

1. $=$ (numerical equality) via $\leq$ and $\geq$
2. $=$ (set equality) via $\subseteq$ and $\supseteq$

$S = \{1, 2, 3\}$, $T = \{2, 3, 1\}$, $U = \{3, 2\}$

	S		T		U	
S	$\subseteq$	$\subset$	$\subseteq$	$\subset$	$\subseteq$	$\subset$
T	$\subseteq$	$\subset$	$\subseteq$	$\subset$	$\subseteq$	$\subset$
U	$\subseteq$	$\subset$	$\subseteq$	$\subset$	$\subseteq$	$\subset$

Is set difference ($\setminus$) commutative?

Combinations: Formula and Interpretation

Power Set

Calculate the power set of {1, 2, 3}.

Given a set S, formulate the cardinality of its power set.

Cardinality of Power Set: Interpreting Formula

- Calculate by considering subsets of various cardinalities.
- Calculate by considering whether a member should be included.

Set of Tuples

Given n sets $S_1, S_2, \ldots, S_n$, a ***cross/Cartesian product*** of theses sets is a set of n-tuples.

Each ***n-tuple*** $(e_1, e_2, \ldots, e_n)$ contains n elements, each of which a member of the corresponding set.

$$S_1 \times S_2 \times \cdots \times S_n = \{(e_1, e_2, \ldots, e_n) \mid e_i \in S_i \land 1 \leq i \leq n\}$$

Example: Calculate {a, b} ✘ {2, 4} ✘ {$, &}

Set of Possible Relations

- Set of possible relations on S and T:
- Dedicated symbol for set of possible relations on S and T:
- Declare that set r is a relation on S and T:

Example: Enumerate **all** relations on {a, b} and {2, 4}.

Hint: How many?